Metric Learning for Hyperspectral Image Segmentation

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Application: Superpixel Segmentation [Thompson et al., 2010]

- Find spatially contiguous, spectrally homogeneous regions (“superpixels”) corresponding to physical features
- Reduces processing time of subsequent analyses
- Yields theoretical noise improvement of order $n^{1/2}$ for a superpixel of size $n$
Graph-based Segmentation Algorithm [Felzenszwalb]

- Image = 8-connected graph weighted by distances $d(x_i, x_j)$ between adjacent pixels $x_i$ and $x_j$
- Agglomerative clustering iteratively connects segments by growing minimum spanning trees

Segment merging criterion:
\[
\text{Dif}(S_a, S_b) > \text{MInt}(S_a, S_b) = \min \left( \text{Int}(S_a) + \frac{k}{|S_a|}, \text{Int}(S_b) + \frac{k}{|S_b|} \right)
\]
- Small $k$ = many segments, large $k$ = few segments, dependant on $d(x_i, x_j)$
Metric Learning for Hyperspectral Image Segmentation

- Segmentation quality dependent on robustness of spectral similarity measure

Unweighted metrics (e.g. Euclidean distance) can be sensitive to noise

Learned metrics can emphasize spectral bands relevant to training classes

Image: CRISM FRT000863e
Mahalanobis Metric Learning

• Goal: learn a task-specific Mahalanobis metric given labeled data
\[ \{x_i, y_i\}_{i=1}^{N} \quad x_i \in \mathbb{R}^d, \ y_i \in [1, k] \]

\[
\begin{align*}
\text{d}_{\text{Euc}}(x_i, x_j)^2 &= (x_i - x_j)^T(x_i - x_j) \\
\text{d}_M(x_i, x_j)^2 &= (x_i - x_j)^T M(x_i - x_j)
\end{align*}
\]

• \( M = AA^T \) = positive semi-definite transformation matrix

• Squashes uninformative / emphasizes informative dimensions w.r.t. classes

Image credit: Weinberger et al. NIPS 2010
Multiclass Linear Discriminant Analysis [Fisher. 1934]

- Maximize separation ratio \( S = \frac{\alpha^T \Sigma_b \alpha}{\alpha^T \Sigma_w \alpha} \), where:
  \[
  \Sigma_w = \frac{1}{NK} \sum_{i=1}^{K} \sum_{j=1}^{N} (x_j - \mu_i)(x_j - \mu_i)^T, \quad \mu_i = \mathbb{E}[x_j | y_j = i]
  \]
  \[
  \Sigma_b = \frac{1}{K} \sum_{i=1}^{K} (\mu_i - \mu)(\mu_i - \mu)^T, \quad \mu = \mathbb{E}[\mu_i]
  \]
- \( S \) maximized when \( \alpha \) the top eigenvector of \( \Sigma_w^{-1} \Sigma_b \)
- \( A = \) top (k-1) eigenvectors of \( \Sigma_w^{-1} \Sigma_b \)
- To prevent degenerate solutions, regularize:
  \[
  \Sigma_w = (1 - \gamma) \Sigma_w + \gamma I, \quad \gamma \in [0, 1]
  \]
Information Theoretic Metric Learning (ITML) [Davis et al. 2007]

- Bijection between Mahalanobis distances and multivariate Gaussians
  \[ \mathcal{N}(x|\mu, \mathbf{M}) = \frac{1}{Z} \exp\left(-\frac{1}{2}d_{\mathbf{M}}(x, \mu)\right) \] (assume fixed \( \mu \))

- Solve:
  \[ \min_{\mathbf{M}} \int \mathcal{N}(x|\mu, \mathbf{M}) \log \left( \frac{\mathcal{N}(x|\mu, \mathbf{M}_0)}{\mathcal{N}(x|\mu, \mathbf{M})} \right) dx \]
  
  \( \mathbf{M}_0 = \) regularization term - known, well-behaved Mahalanobis matrix (e.g., identity or sample covariance matrix)

- Subject to \( \mathbf{M} \succeq 0 \) and pairwise similarity/dissimilarity constraints:
  \[ d_{\mathbf{M}}(x_i, x_j) \leq u \rightarrow (i, j) \in S \]
  \[ d_{\mathbf{M}}(x_i, x_j) \geq l \rightarrow (i, j) \in D \]
Evaluation Methodology

- # of segments (for a fixed image) dependant on (1) similarity metric and (2) segmentation parameter $k$
  - Vary $k$ to compare segmentation maps with similar # of segments for each measure
Case Study: CRISM Imagery

• Images: FRT 3e12, 3fb9, 863e
• 231 bands in [1.06, 2.58] µm
• Class maps defined and verified by geologist using ENVI Spectral Angle Mapper
• Unlabeled materials excluded from performance analysis
Evaluation Measures

- For a set of classes $C$ and a set of segments $S$

- $H(C|S) = \sum_{c \in C, s \in S} p(c, s) \log \frac{p(c)}{p(c, s)}$

  - Measures remaining uncertainty in class map given segmentation partitions
  - If segmentation reproduces class map, $H(C|S) = 0$

- purity$(S, C) = \sum_{s \in S} \frac{\text{pure}(s, C)}{|s|}$

  - pure$(s, C) = 1$ if all pixels in segment $s$ belong to a single class $c$ in $C$
Image 863e Segmentation Results
# Image 863e Segment Purity Scores

<table>
<thead>
<tr>
<th>Class (# pixels)</th>
<th>Euc</th>
<th>LDA</th>
<th>ITML</th>
</tr>
</thead>
<tbody>
<tr>
<td>FeMg Smectite (6443)</td>
<td>26</td>
<td>49</td>
<td>48</td>
</tr>
<tr>
<td>Kaolinite (4051)</td>
<td>98</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>Montmorillonite (10901)</td>
<td>11</td>
<td>31</td>
<td>17</td>
</tr>
<tr>
<td>Nontronite (4753)</td>
<td>37</td>
<td>52</td>
<td>40</td>
</tr>
<tr>
<td>Neutral Region (115225)</td>
<td>97</td>
<td>99</td>
<td>98</td>
</tr>
<tr>
<td><strong>Average (141373)</strong></td>
<td><strong>53</strong></td>
<td><strong>66</strong></td>
<td><strong>60</strong></td>
</tr>
</tbody>
</table>
$H(C|S)$ Results: Images 3e12, 3fb9, 863e
Conclusions / Future Work

• Learned metrics can significantly improve the quality of hyperspectral segmentation results

• Simple techniques (e.g., LDA) with only a few training samples often outperform more computationally expensive metric learning methods
  
  • Additional samples may improve ITML accuracy

• Future work: comparison to additional metric learning algorithms (neighborhood components analysis, relevant components analysis), and transfer learning scenarios
  
  • Initial results indicate LDA competitive with state of the art Mahalanobis metric learning algorithms
HiiHAT IDL/ENVI Toolkit
http://hyperspectral.jpl.nasa.gov

- ENVI Toolkit for hyperspectral image analysis
- Includes superpixel segmentation, endmember detection, feature enhancement and metric learning functions
- Free, non-commercial research licenses available
References

• Ronald Fisher (1936) The Use of Multiple Measurements in Taxonomic Problems In: Annals of Eugenics, 7, p. 179--188


